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MATH 204
Assignment 23

Section 4.4

21. $P_{\mathcal{B}}\vec{x} = [\vec{x}]_{\mathcal{B}} \Rightarrow P_{\mathcal{B}}^{-1}P_{\mathcal{B}}\vec{x} = P_{\mathcal{B}}^{-1}[\vec{x}]_{\mathcal{B}} \Rightarrow \vec{x} = P_{\mathcal{B}}^{-1}[\vec{x}]_{\mathcal{B}}$

    $P_{\mathcal{B}} = \begin{bmatrix} 1 & -2 \\ -4 & 9 \end{bmatrix} \Rightarrow P_{\mathcal{B}}^{-1} = \begin{bmatrix} 9 & 2 \\ 4 & 1 \end{bmatrix} = A$

35. Let $[P_{\mathcal{B}}|\vec{x}] = \left[\begin{array}{cc|c} 11 & 14 & 19 \\ -5 & -8 & -13 \\ 10 & 13 & 18 \\ 7 & 10 & 15 \end{array}\right] \xrightarrow{rref} \left[\begin{array}{cc|c} 1 & 0 & \frac{-5}{3} \\ 0 & 1 & \frac{8}{3} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow \vec{x} = \begin{bmatrix} \frac{-5}{3} \\ \frac{8}{3} \end{bmatrix}$

37. $P_{\mathcal{B}} = \begin{bmatrix} \vec{b_1} & \vec{b_2} & \vec{b_3} \end{bmatrix} = \begin{bmatrix} 2.6 & 0 & 0 \\ -1.5 & 3 & 0 \\ 0 & 0 & 4.8 \end{bmatrix} \Rightarrow P_{\mathcal{B}} \begin{bmatrix} \frac{1}{2} \\ \frac{1}{4} \\ \frac{1}{6} \end{bmatrix}_{\mathcal{B}} = \begin{bmatrix} 1.3 \\ 0 \\ .8 \end{bmatrix} = \vec{x}$

38. $P_{\mathcal{B}} = \begin{bmatrix} \vec{b_1} & \vec{b_2} & \vec{b_3} \end{bmatrix} = \begin{bmatrix} 2.6 & 0 & 0 \\ -1.5 & 3 & 0 \\ 0 & 0 & 4.8 \end{bmatrix} \Rightarrow P_{\mathcal{B}} \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{3} \end{bmatrix}_{\mathcal{B}} = \begin{bmatrix} 1.3 \\ .75 \\ 1.6 \end{bmatrix} = \vec{x}$

Section 4.5

1. Basis $= \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix} \right\}$, dim $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix} \right\} = 2$

    Bases are sets of linearly independent vectors therefore their dim is equal to the number of vectors in the set.

2. Basis $= \left\{ \begin{bmatrix} 4 \\ -3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$, dim $\left\{ \begin{bmatrix} 4 \\ -3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\} = 2$

3. Basis $= \left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -3 \\ 0 \end{bmatrix} \right\}$, dim $\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -3 \\ 0 \end{bmatrix} \right\} = 3$

10. Let $A = \begin{bmatrix} 2 & -4 & -3 \\ -5 & 10 & 6 \end{bmatrix}$. $A \xrightarrow{rref} \begin{bmatrix} 1 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Since there are 2 pivots $\dim \operatorname{Col} A = 2$.

11. Let $A = \begin{bmatrix} 1 & 3 & 9 & 7 \\ 0 & 1 & 4 & -3 \\ 2 & 1 & -2 & 1 \end{bmatrix}$. $A \xrightarrow{rref} \begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & 4 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. Since there are 2 pivots $\dim \operatorname{Col} A = 2$.

13. Since 2 columns are pivotless $\dim \mathrm{Nul}A = 2$. Since 3 columns have pivots $\dim \mathrm{Col}A = 3$.

16. $A = \begin{bmatrix} 3 & 4 \\ -6 & 10 \end{bmatrix}$. $A \xrightarrow{rref} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Since there are 2 pivots $\dim \mathrm{Col}A = 2$. Since there are 0 pivotless columns $\dim \mathrm{Nul}A = 0$.

Section 4.6

1. $\mathrm{Rank}A = \dim \mathrm{Col}A = 2,\ \dim \mathrm{Nul}A = 2,\ \text{Basis Col}A = \left\{ \begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ -6 \end{bmatrix} \right\}$,

   $\text{Basis Nul}A = \left\{ \begin{bmatrix} 1 \\ -5 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ -6 \\ 0 \\ 1 \end{bmatrix} \right\}$, $\text{Basis Row}A = \{(1,0,-1,5),(0,-2,5,-6)\}$

2. $\text{Rank } A = 3,\ \dim \mathrm{Col}A = 2,\ \text{Basis Col}A = \left\{ \begin{bmatrix} 1 \\ -2 \\ -3 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ -6 \\ -6 \\ 4 \end{bmatrix}, \begin{bmatrix} 9 \\ -10 \\ -3 \\ 0 \end{bmatrix} \right\}$

   $\text{Basis Nul}A = \left\{ \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ 3 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}$, $\text{Basis Row}A = \{(1,-3,0,5,-7),(0,0,2,-3,8),(0,0,0,0,5)\}$

5. $\text{Rank } A = \dim \mathrm{Col}A = 3 =$ number of pivot columns. $\dim \mathrm{Nul}A = 8 - 3 = 5$. $\dim \mathrm{Row}A = 3$. $\text{Rank } A^T = \dim \mathrm{Col}A = 3$

6. $\text{Rank } A = \dim \mathrm{Col}A = 3 =$ number of pivot columns. $\dim \mathrm{Nul}A = 0$. $\dim \mathrm{Row}A = 3$. Rank $A^T = \dim \mathrm{Col}A = 3$

7. Col$A$ is made up of 4 linearly independent vectors in $\mathbb{R}^4$. Therefore Col$A$ spans all of $\mathbb{R}^4$. There are 3 pivotless columns in A, so Nul$A$ will span a 3 dimensional subspace of $\mathbb{R}^7$. $\mathrm{Nul}A \neq \mathbb{R}^3$.

9. Null space of a 5×6 matrix is a 4 dimensional subspace, therefore A has 4 pivotless columns. $\therefore$ there are 2 columns with pivots in A making $\dim \mathrm{Col}A = 2$.

13. If A is a 7×5 matrix the largest amount of possible pivots is 5. $\therefore$ the maximum Rank$A$ could be is 5.

    If A is a 5×7 matrix the largest amount of possible pivots is 5. $\therefore$ the maximum Rank$A$ could be is 5.