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MATH 204

Assignment 22

Section 4.3

6. Cannot be a basis for $\mathbb{R}^3$. Is a set of linearly independent vectors which could form a basis for a 2 dimensional subspace of $\mathbb{R}^3$.

11. $x = -2y - z \Rightarrow x = y \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + z \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \Rightarrow$. Basis $= \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

16. Let $A_{16} = \begin{bmatrix} 1 & -2 & 6 & 5 & 0 \\ 0 & 1 & -1 & -3 & 3 \\ 0 & -1 & 2 & 3 & -1 \\ 1 & 1 & -1 & -4 & 1 \end{bmatrix} \xrightarrow{rref} \begin{bmatrix} 1 & 0 & 0 & -1 & -2 \\ 0 & 1 & 0 & -3 & 5 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} x_4 + 2x_5 \\ 3x_4 - 5x_5 \\ -2x_5 \\ x_4 \\ x_5 \end{bmatrix}$

$= x_4 \begin{bmatrix} 1 \\ 3 \\ 0 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 2 \\ -5 \\ -2 \\ 0 \\ 1 \end{bmatrix}$. Basis Col $A_{16} = \begin{bmatrix} 1 \\ 3 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -5 \\ -2 \\ 0 \\ 1 \end{bmatrix}$

24. To be a basis a set of vectors needs to have two properties. The first is that the set of vectors is linearly independent. The second is that the subspace spans the space it is supposed to be spanning. $\mathcal{B}$ has both of these properties, we are given it is linearly independent in $\mathbb{R}^n$ and because it is linearly independent a coefficient matrix with column vectors that are in the set must have a pivot in every column. That means the matrix must have n pivots, which means it spans at least a n dimensional subspace of $\mathbb{R}^m$.

25. The set of vectors is not a basis for H because the set of vectors spans all of $\mathbb{R}^3$ not just the subset of $\mathbb{R}^3$ defined in the problem.

29. If there are k vectors in a set and a matrix is made out of them, there can be at most k pivots because that is the number of vectors possible. If $k < n$ there cannot be a pivot in every row. Therefore the set of vectors cannot span all of $\mathbb{R}^n$ and the set of vectors cannot be a basis for $\mathbb{R}^n$.

30. If the number of vectors is greater than the number of entries per vector the set of vectors cannot be linearly independent. Therefore the set of vectors cannot be a basis.

Section 4.4

2. $\vec{x} = 8 \begin{bmatrix} 4 \\ 5 \end{bmatrix} - 5 \begin{bmatrix} 6 \\ 7 \end{bmatrix} = \begin{bmatrix} -10 \\ 5 \end{bmatrix}$

4. $\vec{x} = -4\begin{bmatrix}-1\\2\\0\end{bmatrix} + 8\begin{bmatrix}3\\-5\\2\end{bmatrix} - 7\begin{bmatrix}4\\-7\\3\end{bmatrix} = \begin{bmatrix}0\\1\\-5\end{bmatrix}$

5. $\left[\begin{array}{cc|c}1 & 2 & -2\\-3 & -5 & 1\end{array}\right] \xrightarrow{rref} \left[\begin{array}{cc|c}1 & 0 & 8\\0 & 1 & -5\end{array}\right] \Rightarrow 8\begin{bmatrix}1\\-3\end{bmatrix} - 5\begin{bmatrix}2\\-5\end{bmatrix} = \begin{bmatrix}-2\\1\end{bmatrix} \Rightarrow [\vec{x}]_{\mathcal{B}} = \begin{bmatrix}8\\-5\end{bmatrix}$

9. Transform $\vec{e_1}$ and $\vec{e_2}$,

Let $P_{\mathcal{B}} = \begin{bmatrix}2 & 1\\-9 & 8\end{bmatrix}$ $\quad \begin{bmatrix}2 & 1\\-9 & 8\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}2\\-9\end{bmatrix}$ $\quad \begin{bmatrix}2 & 1\\-9 & 8\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}1\\8\end{bmatrix}$

10. Transform $\vec{e_1}$ and $\vec{e_2}$ and $\vec{e_3}$,

Let $P_{\mathcal{B}} = \begin{bmatrix}3 & 2 & 8\\-1 & 0 & -2\\4 & -5 & 7\end{bmatrix}$ $\quad \begin{bmatrix}3 & 2 & 8\\-1 & 0 & -2\\4 & -5 & 7\end{bmatrix}\begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{bmatrix}3\\-1\\4\end{bmatrix}$ $\quad \begin{bmatrix}3 & 2 & 8\\-1 & 0 & -2\\4 & -5 & 7\end{bmatrix}\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}2\\0\\-5\end{bmatrix}$

$\begin{bmatrix}3 & 2 & 8\\-1 & 0 & -2\\4 & -5 & 7\end{bmatrix}\begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}8\\-2\\7\end{bmatrix}$

11. Let $P_{\mathcal{B}_{11}} = \begin{bmatrix}3 & -4\\-5 & 6\end{bmatrix} \Rightarrow {P_{\mathcal{B}_{11}}}^{-1} = \begin{bmatrix}-3 & -2\\-2.5 & -1.5\end{bmatrix} \Rightarrow {P_{\mathcal{B}_{11}}}^{-1}\vec{x} = [\vec{x}]_{\mathcal{B}} = \begin{bmatrix}-3 & -2\\-2.5 & -1.5\end{bmatrix}\begin{bmatrix}2\\-6\end{bmatrix} = \begin{bmatrix}6\\4\end{bmatrix}$

12. Let $P_{\mathcal{B}_{12}} = \begin{bmatrix}4 & 6\\5 & 7\end{bmatrix} \Rightarrow {P_{\mathcal{B}_{12}}}^{-1} = \frac{1}{2}\begin{bmatrix}-7 & 6\\5 & -4\end{bmatrix} \Rightarrow {P_{\mathcal{B}_{12}}}^{-1}\vec{x} = [\vec{x}]_{\mathcal{B}} = \frac{1}{2}\begin{bmatrix}-7 & 6\\5 & -4\end{bmatrix}\begin{bmatrix}2\\0\end{bmatrix} = \begin{bmatrix}-7\\5\end{bmatrix}$

17. $\left[\begin{array}{ccc|c}1 & 2 & -3 & 1\\-3 & -8 & 7 & 1\end{array}\right] \xrightarrow{rref} \left[\begin{array}{ccc|c}1 & 0 & -5 & 5\\0 & 1 & 1 & -2\end{array}\right] \Rightarrow \begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}5x_3 + 5\\-x_3 - 2\\x_3\end{bmatrix} \Rightarrow x_3\begin{bmatrix}5\\-1\\1\end{bmatrix} + \begin{bmatrix}5\\-2\\0\end{bmatrix}$

Two such vectors are $\begin{bmatrix}10\\-3\\1\end{bmatrix}$ and $\begin{bmatrix}20\\-6\\2\end{bmatrix}$