Kyle Daling

MATH 204

Assignment 21

Section 4.2

10. W is null space of matrix A where $A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

12. W is not a subspace because it does not contain the zero vector. It is impossible for $d = 0$ and $d = -.5$ which is necessary for rows 3 and 4 to have zero entries.

18. (a) Nul A is a 1 dimensional subspace of $\mathbb{R}^3$.

    (b) $\begin{bmatrix} 7 & -2 & 0 \\ -2 & 0 & -5 \\ 0 & -5 & 7 \\ -5 & 7 & -2 \end{bmatrix} \xrightarrow{rref} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$. Col A is a 3 dimensional subspace of $\mathbb{R}^4$.

23. $\left[\begin{array}{cc|c} -6 & 12 & 2 \\ -3 & 6 & 1 \end{array}\right] \xrightarrow{rref} \left[\begin{array}{cc|c} 1 & -2 & \frac{-1}{3} \\ 0 & 0 & 0 \end{array}\right]$. $\vec{w}$ is in Col A because the matrix is consistent.

    $2\begin{bmatrix} -6 \\ -3 \end{bmatrix} + 1\begin{bmatrix} 12 \\ 6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

24. $\left[\begin{array}{ccc|c} -8 & -2 & -9 & 2 \\ 6 & 4 & 8 & 1 \\ 4 & 0 & 4 & -2 \end{array}\right] \xrightarrow{rref} \left[\begin{array}{ccc|c} 1 & 0 & 1 & \frac{-1}{2} \\ 0 & 1 & \frac{1}{2} & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$. $\vec{w}$ is in Col A because the matrix is consistent.

    $2\begin{bmatrix} -8 \\ 6 \\ 4 \end{bmatrix} + 1\begin{bmatrix} -2 \\ 4 \\ 0 \end{bmatrix} - 2\begin{bmatrix} -9 \\ 8 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. $\vec{w}$ is in Nul A.

27. Let A_{27} be the matrix that solves the system given. If $\begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$ is one vector in Nul A_{27}, and $\begin{bmatrix} 30 \\ 20 \\ -10 \end{bmatrix}$ is another vector also in Null A_{27}, then A_{27} must have one pivotless column. This allows Nul A_{27} to span a line through the origin with both of those solutions on it.

28. Let the systems be represented by $A\vec{x} = \vec{s_1}$ and $A\vec{x} = \vec{s_2}$. If $A\vec{x}$ is consistent for $\vec{s_1}$ then $\vec{s_1}$ is in Col A. Since $\vec{s_2} = 5\vec{s_1}$ it is also in Col A. Therefore both systems have solutions.

Section 4.3

1. The set of vectors in problem 1 are a basis for $\mathbb{R}^3$.

4. $\begin{bmatrix} 2 & 1 & -7 \\ -2 & -3 & 5 \\ 1 & 2 & 4 \end{bmatrix} \xrightarrow{rref} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. The set of vectors span all of $\mathbb{R}^3$. They must be a basis.

7. The set of vectors is linearly dependent because the vectors are not scalar multiples of one another. The two vectors do not span all of $\mathbb{R}^3$ because there can only be two pivots.

9. First I must identify the pivots. To do this I will row reduce.
$\begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & -5 & 4 \\ 3 & -2 & 1 & -2 \end{bmatrix} \xrightarrow{rref} \begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & -5 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. $x_1 = 3x_3 - 2x_4$ and $x_2 = 5x_3 - 4x_4$ and $x_3 = x_3$ and $x_4 = x_4$.

   Basis $= \begin{bmatrix} 3 \\ 5 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ -4 \\ 0 \\ 1 \end{bmatrix}$

12. $\begin{bmatrix} 1 & \frac{1}{5} \\ 0 & 0 \end{bmatrix}$. $x_1 = \frac{-1}{5}x_2$ and $x_2 = x_2$. Basis Nul $= \begin{bmatrix} \frac{-1}{5} \\ 1 \end{bmatrix}$

13. $A \xrightarrow{rref} \begin{bmatrix} 1 & 0 & 6 & 5 \\ 0 & 1 & \frac{5}{2} & \frac{3}{2} \\ 0 & 0 & 0 & 0 \end{bmatrix}$. $x_1 = -6x_3 - 5x_4$ and $x_2 = \frac{-5}{2}x_3 + \frac{-3}{2}x_4$ and $x_3 = x_3$ and $x_4 = x_4$

   Basis Nul $A = \begin{bmatrix} -6 \\ \frac{-5}{2} \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ \frac{-3}{2} \\ 0 \\ 1 \end{bmatrix}$. To find Basis Col $A = \begin{bmatrix} -2 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} 4 \\ -6 \\ 8 \end{bmatrix}$ remove the columns that don't have pivots.

14. Basis Col $A = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 8 \\ -9 \\ 0 \end{bmatrix}$. Basis Nul $A = \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ \frac{7}{5} \\ 0 \\ 1 \\ 0 \end{bmatrix}$

15. Let $A = \begin{bmatrix} 1 & 0 & -3 & 1 & 2 \\ 0 & 1 & -4 & -3 & 1 \\ -3 & 2 & 1 & -8 & -6 \\ 2 & -3 & 6 & 7 & 9 \end{bmatrix} \xrightarrow{rref} \begin{bmatrix} 1 & 0 & -3 & 0 & 4 \\ 0 & 1 & -4 & 0 & -5 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$. $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 3x_3 - 4x_5 \\ 4x_3 + 5x_5 \\ x_3 \\ 2x_5 \\ x_5 \end{bmatrix}$

   Basis Col $A = \begin{bmatrix} 1 \\ 0 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ -8 \\ 7 \end{bmatrix}$

20. Since the solution to the homogeneous system isn't the trivial solution I know that there must be at least one free variable in the system. Therefore the minimal basis must be $\begin{bmatrix} 7 \\ 4 \\ -9 \\ -5 \end{bmatrix}, \begin{bmatrix} 4 \\ -7 \\ 2 \\ 5 \end{bmatrix}$

Section 4.4

1. $[\vec{x}]_{\mathcal{B}} = 5 \begin{bmatrix} 3 \\ -5 \end{bmatrix} + 3 \begin{bmatrix} -4 \\ 6 \end{bmatrix} = \begin{bmatrix} 3 \\ -7 \end{bmatrix}$

3. $[\vec{x}]_{\mathcal{B}} = 3 \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix} + 0 - 1 \begin{bmatrix} 4 \\ -7 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ -5 \\ 9 \end{bmatrix}$

6. $\left[\begin{array}{cc|c} 1 & 5 & 4 \\ -2 & -6 & 0 \end{array}\right] \xrightarrow{rref} \left[\begin{array}{cc|c} 1 & 0 & -6 \\ 0 & 1 & 2 \end{array}\right]$. $[\vec{x}]_{\mathcal{B}} = \begin{bmatrix} -6 \\ 2 \end{bmatrix}$

7. $\left[\begin{array}{ccc|c} 1 & -3 & 2 & 8 \\ -1 & 4 & -2 & -9 \\ -3 & 9 & 4 & 6 \end{array}\right] \xrightarrow{rref} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right]$. $[\vec{x}]_{\mathcal{B}} = \begin{bmatrix} -1 \\ -1 \\ 3 \end{bmatrix}$