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MATH 204
Assignment 20

Section 3.2

19. It appears that two row operations were performed on R_2. The first was $2R_2 \to R_2$. The second was $R_1 + R_2 \to R_2$. This would change the determinate by 2(1) so $7 \times 2 = 14$.

33. Since A and B are square, we know that $\det AB = (\det A)(\det B)$. We know that both $\det A$ and $\det B$ are scalar numbers so the order they are in doesn't matter. Therefore $\det BA = (\det B)(\det A) = (\det A)(\det B) = \det AB$.

40. a. $\det AB = (\det A)(\det B) = (-1)(2) = -2$
    b. $\det B^5 = \det BBBBB = (\det B)(\det B)(\det B)(\det B)(\det B) = 10$
    c. $\det 2A = 2^4 \det A = (16)(-1) = -16$
    d. $\det A^T A = (\det A^T)(\det A) = (\det A)(\det A) = (-1)(-1) = 1$
    e. $\det B^{-1}AB = (\det B^{-1})(\det A)(\det B) = (2)(-1)(2) = -4$

Supplementary Problems Chapter 3

1. (a) True, if a matrix has a zero determinant then it cannot be invertible. By the invertible matrix theorem that means the matrix cannot have a pivot in every column. Therefore the column vectors of the matrix must be linearly dependent and one column must be a multiple of another.
   (b) True, if two rows of a matrix are the same that means the matrix doesn't have a pivot in every row. If the 3×3 square matrix doesn't have 3 pivots that means the matrix isn't invertible. That means that the determinant of the matrix is zero.
   (c) False, $\det 5A$ is equivalent to $\det AAAAA$. $5 \det A$ is equavalent to matrix A with one row scaled by 5. $\det 5A \neq 5 \det A$.
   (d) False, that is not a valid property of determinants.
   (e) False, if $\det A = 2$ then $\det AAA = (\det A)^3 = 2^3$.
   (f) False, row interchange scales the determinant by a factor of -1. $\det B = -\det A$.
   (g) True, scaling a row by 5 also scales the determinant by 5. $\det B = 5 \det A$.
   (h) True, linear combinations don't affect the determinant.
   (i) False, the determinant of a transposed matrix is the same as the determinant of the original matrix. If the matrices in the problem aren't square, then they don't have determinants.
   (j) False, this is scaling all the rows in a matrix by -1. $\det -A = (-1)^n \det A$ where A is a $n \times n$ matrix.

(k) True,

(m) False, the determinant gives the area of a parallelogram, not a triangle.

(o) False, the determinant of the inverse of a matrix is the inverse of the determinant of the matrix. $\det A^{-1} = \frac{1}{\det A}$.

(p) True, the determinant of the inverse of a matrix is the inverse of the determinant of the matrix. When multiplied together the determinant would be one. Also the determinant of an identity matrix is 1, which is what one would get when one would multiply a matrix by its inverse.

5. $\begin{vmatrix} 9 & 1 & 9 & 9 & 9 \\ 9 & 0 & 9 & 9 & 2 \\ 4 & 0 & 0 & 5 & 0 \\ 9 & 0 & 3 & 9 & 0 \\ 6 & 0 & 0 & 7 & 0 \end{vmatrix} = 1 \begin{vmatrix} 9 & 9 & 2 & 9 \\ 0 & 5 & 0 & 4 \\ 3 & 9 & 0 & 9 \\ 0 & 7 & 0 & 6 \end{vmatrix} = 1 \left(2 \begin{vmatrix} 4 & 0 & 5 \\ 9 & 3 & 9 \\ 6 & 0 & 7 \end{vmatrix} \right) = 1 \left(2 \left(3 \begin{vmatrix} 7 & 6 \\ 5 & 4 \end{vmatrix} \right) \right) =$

$1 \times 2 \times 3 \times (7 \times 4 - 5 \times 6) = -12$

Section 4.1

12. $s \begin{bmatrix} 1 \\ 1 \\ 2 \\ 0 \end{bmatrix} + t \begin{bmatrix} 3 \\ -1 \\ -1 \\ 4 \end{bmatrix}$. $W = \text{Span} \left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ -1 \\ 4 \end{bmatrix} \right\}$

21. Well, it contains the zero matrix is a, b, d are all 0.

    Scalar multiples of the matrix are still of the form of the matrix.

    Adding two matrices of the same form gives a matrix in the same form.

    Since all the above hold, I conclude that this matrix spans a subspace.

Section 4.2

3. $A\vec{x} = \vec{0} \Rightarrow \left[\begin{array}{cccc|c} 1 & 3 & 5 & 0 & 0 \\ 0 & 1 & 4 & -2 & 0 \end{array}\right] \xrightarrow{rref} \left[\begin{array}{cccc|c} 1 & 0 & -7 & 6 & 0 \\ 0 & 1 & 4 & -2 & 0 \end{array}\right] \therefore x_1 = 7x_3 - 6x_4$ and $x_2 = -4x_3 + 2x_4$

   $\therefore$

8. W is not a vector space because it does not contain the zero vector.

15. Linear combinations of $r \begin{bmatrix} 0 \\ 1 \\ 4 \\ 3 \end{bmatrix} + s \begin{bmatrix} 2 \\ 1 \\ 1 \\ -1 \end{bmatrix} + t \begin{bmatrix} 3 \\ -2 \\ 0 \\ -1 \end{bmatrix} \Rightarrow A = \begin{bmatrix} 0 & 2 & 3 \\ 1 & 1 & -2 \\ 4 & 1 & 0 \\ 3 & -1 & -1 \end{bmatrix} \begin{bmatrix} r \\ s \\ t \end{bmatrix}$

17. For $A\vec{x} = \vec{0}$ to have solutions $\vec{x}$ must be a vector in $\mathbb{R}^2$.

21. $\begin{bmatrix} 2 & -6 \\ -1 & 3 \\ -4 & 12 \\ 3 & -9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \to \left[\begin{array}{cc|c} 2 & -6 & 0 \\ -1 & 3 & 0 \\ -4 & 12 & 0 \\ 3 & -9 & 0 \end{array}\right] \xrightarrow{rref} \left[\begin{array}{cc|c} 1 & -3 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right] \therefore x_1 = 3x_2 \therefore \begin{bmatrix} 3 \\ 1 \end{bmatrix} \in \text{Nul}A$

$$\begin{bmatrix} 2 \\ -1 \\ -4 \\ 3 \end{bmatrix} + \begin{bmatrix} -6 \\ 3 \\ 12 \\ -9 \end{bmatrix} = \begin{bmatrix} -4 \\ 2 \\ 8 \\ -6 \end{bmatrix} \in \text{Col}A$$

22. $\left[\begin{array}{cccc|c} 1 & 3 & 5 & 0 & 0 \\ 0 & 1 & 4 & -2 & 0 \end{array}\right] \xrightarrow{rref} \left[\begin{array}{cccc|c} 1 & 0 & -7 & 6 & 0 \\ 0 & 1 & 4 & -2 & 0 \end{array}\right] \therefore x_1 - 7x_3 + 6x_4 = 0$ and $x_2 + 4x_3 - 2x_4 = 0$

$$\begin{bmatrix} 7 \\ -4 \\ 1 \\ 0 \end{bmatrix} \in \text{Nul}A$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 3 \\ 1 \end{bmatrix} + \begin{bmatrix} 5 \\ 4 \end{bmatrix} + \begin{bmatrix} 0 \\ -2 \end{bmatrix} = \begin{bmatrix} 9 \\ 3 \end{bmatrix} \in \text{Col } A$$

Section 4.3

2. The set is linearly dependent because the zero vector is an entry in the set. The set of vectors can only span a 2 dimensional subspace of $\mathbb{R}^3$.

5. The set is linearly dependent because the zero vector is an entry in the set. However there is a pivot in every column so the set of vectors will span all of $\mathbb{R}^3$.