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MATH 204

Assignment 19

Section 3.1

35. $EA = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a+ck & b+dk \\ c & d \end{bmatrix} \rightarrow \begin{vmatrix} a+ck & b+dk \\ c & d \end{vmatrix} = d(a+ck) - c(b+dk) = ad + cdk - cb - cdk = ad - cb$

   $\begin{vmatrix} 1 & k \\ 0 & 1 \end{vmatrix} = 1$ and $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$ ✓

37. $A = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$ $\qquad 5A = \begin{bmatrix} 15 & 5 \\ 20 & 10 \end{bmatrix}$ $\qquad 5\begin{vmatrix} 3 & 1 \\ 4 & 2 \end{vmatrix} = 5(6-4) = 10 \neq 150 - 100 = \begin{vmatrix} 15 & 5 \\ 20 & 10 \end{vmatrix}$

38. $\det(kA) = \det\left(k\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{vmatrix} ka & kb \\ kc & kd \end{vmatrix} = ka(kd) - kb(kc) = k^2ad - k^2bc = k^2(ad-bc) = k^2 \det(A)$

Section 3.2

3. This demonstrates the row replacement property of determinants. When a row replacement is performed the value of the determinant does not change.

5. $\begin{bmatrix} 1 & 5 & -6 \\ -1 & -4 & 4 \\ -2 & -7 & 9 \end{bmatrix} \xrightarrow{R_1+R_2 \to R_2} \begin{bmatrix} 1 & 5 & -6 \\ 0 & 1 & -2 \\ -2 & -7 & 9 \end{bmatrix} \xrightarrow{2R_1+R_3 \to R_3} \begin{bmatrix} 1 & 5 & -6 \\ 0 & 1 & -2 \\ 0 & 3 & -3 \end{bmatrix} \xrightarrow{-3R_2+R_3 \to R_3}$

   $\begin{bmatrix} 1 & 5 & -6 \\ 0 & 1 & -2 \\ 0 & 0 & 3 \end{bmatrix} \Rightarrow \begin{vmatrix} 1 & 5 & -6 \\ 0 & 1 & -2 \\ 0 & 0 & 3 \end{vmatrix} = 1 \times 1 \times 3 = 3$

15. It appears that this matrix has had its third row scaled by 5. This would similarly scale the value of the determinant by 5. Therefore $\det M_{15} = 5 \times 7 = 35$

18. It appears that two row replacements were done. $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} d & e & f \\ a & b & c \\ g & h & i \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_3}$

   $\begin{bmatrix} g & h & i \\ a & b & c \\ d & e & f \end{bmatrix}$. Therefore $\det M_{18} = -1 \times -1 \times 7 = 7$.

21. $\begin{vmatrix} 2 & 3 & 0 \\ 1 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix}$ by calculator $= -1$. Since $\det M_{21} \neq 0$ the matrix must be invertible.

26. Let M_{26} be a matrix with column vectors that are in the set given in the problem.
$M_{26} = \begin{bmatrix} 3 & 2 & -2 & 0 \\ 5 & -6 & -1 & 0 \\ -6 & 0 & 3 & 0 \\ 4 & 7 & 0 & -3 \end{bmatrix}$ $\det(M_{26}) = 0$. Since the determinant of the matrix is equal to zero, the matrix must not be invertible. Since the matrix is not invertible the matrix must have less than 4 pivots. Since the matrix has less than 4 pivots it must not have a pivot in every column which makes the set of vectors linearly dependent.

29. $\det(BBBBB) = \det(B)\det(B)\det(B)\det(B)\det(B) = (-2)^5 = -32$.

31. Taking the hint in the back of the book, I am going to prove $(\det A)(\det A^{-1}) = 1$. While proving this for a 2×2 matrix doesn't prove it for all matrices this is at least a good start.
$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ by the definition of matrix inverse as given in the book $A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$
$\det A^{-1} = \frac{1}{ad-bc}(da - cb)$

Section 4.1

2. a. If $\vec{u}$ is in W then $c\vec{u}$ is also in W. Multiplying any vector in the first quadrant by a negative number gives a vector in the third quadrant.

   b. $\vec{u} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\vec{v} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$ $\vec{u} + \vec{v} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$. This violates the condition that $xy \geq 0$ for W.

11. $\vec{u} = \begin{bmatrix} 7 \\ 1 \\ 1 \end{bmatrix}$ $\vec{v} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$. Since W is the Span of two vectors in $\mathbb{R}^3$ it must be a subspace of $\mathbb{R}^2$.

14. Let $[M_{14}|\vec{w}] = \left[\begin{array}{ccc|c} 1 & 2 & 4 & 8 \\ 0 & 1 & 2 & 1 \\ -1 & 3 & 6 & 7 \end{array}\right] \xrightarrow{rref} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$. Since there is a pivot in the augmented column of the matrix defined above, the system cannot be consistent. Therefore $\vec{w}$ is not in Span $\{v_1, v_2, v_3\}$.

16. The matrix can be expressed as $a\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} + b\begin{bmatrix} 0 \\ -6 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$. That system does not contain the zero vector.

18. The set of vectors can be expressed as $a\begin{bmatrix} 4 \\ 0 \\ 1 \\ -2 \end{bmatrix} + b\begin{bmatrix} 3 \\ 0 \\ 1 \\ 0 \end{bmatrix} + c\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$. I will put them into a matrix to determine what they span.

$\begin{bmatrix} 4 & 3 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \\ -2 & 0 & 1 \end{bmatrix} \xrightarrow{rref} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$. The matrix with column vectors as given above spans a 3 dimensional subspace of $\mathbb{R}^4$.

Section 4.2

2. $\begin{bmatrix} 5 & 21 & 19 \\ 13 & 23 & 2 \\ 8 & 14 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ -3 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. The vector $\begin{bmatrix} 5 \\ -3 \\ 2 \end{bmatrix}$ is in Nul A.

7. W is not a vector space because linear combinations of $a+b+c=2$ do not contain the zero vector.

9. W is a vector space because linear combinations of $a \begin{bmatrix} 1 \\ 2 \end{bmatrix} + b \begin{bmatrix} -2 \\ 0 \end{bmatrix} + c \begin{bmatrix} -4 \\ -1 \end{bmatrix} + d \begin{bmatrix} 0 \\ -3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ exist.

13. $\begin{bmatrix} c-6d \\ d \\ c \end{bmatrix} = c \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + d \begin{bmatrix} -6 \\ 1 \\ 0 \end{bmatrix}$. Therefore W can be written as Span $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -6 \\ 1 \\ 0 \end{bmatrix} \right\}$. Therefore W is a vector space.