Kyle Daling

MATH 204

Assignment 18

Section 3.1

19. $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$ and $\begin{vmatrix} c & d \\ a & b \end{vmatrix} = cb - ad = -(ad - cb) \Rightarrow \begin{vmatrix} c & d \\ a & b \end{vmatrix} = -\begin{vmatrix} a & b \\ c & d \end{vmatrix}$

    Swapping two rows changes the sign of the determinant.

20. $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$ and $\begin{vmatrix} a & b \\ kc & kd \end{vmatrix} = kad - kbc = k(ad - bc) \Rightarrow \begin{vmatrix} a & b \\ kc & kd \end{vmatrix} = k\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right)$

    Scaling a row by scalar k scales the determinant of the matrix by k.

21. $\begin{vmatrix} 3 & 4 \\ 5 & 6 \end{vmatrix} = (3 \times 6) - (5 \times 4) = -2$ and $\begin{vmatrix} 3 & 4 \\ 5+3k & 6+4k \end{vmatrix} = 3(6+4k) - 4(5+3k) = 18 + 12k - 20 - 12k = 18 - 20 = -2 \Rightarrow \begin{vmatrix} 3 & 4 \\ 5 & 6 \end{vmatrix} = \begin{vmatrix} 3 & 4 \\ 5+3k & 6+4k \end{vmatrix}$

    Performing a row replacement doesn't affect the value of the determinant.

25. $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & k & 1 \end{vmatrix} = 1\begin{vmatrix} 1 & 0 \\ k & 1 \end{vmatrix} - 0 + 0 = 1 \times 1 = 1$

27. $\begin{vmatrix} k & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = k\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} - 0 + 0 = k(1) = k$

29. $\begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} - 0 + 0 = 1(-1) = -1$

33. $EA = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} c & d \\ a & b \end{bmatrix} \Rightarrow \begin{vmatrix} c & d \\ a & b \end{vmatrix} = cb - ad = -1 \times (ad - cb) = \left(\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}\right)\left(\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right)$

Section 3.2

1. [1] shows the results of a row interchange on the determinant. A row interchange negates the value of the determinant.

2. Scaling a row by a multiple (in this case 2) causes the determinant to be scaled by the same amount.

16. It looks like one row was scaled by 3. This scales the determinant of the matrix by 3 $\Rightarrow$ 3(7)=21.

17. It looks like two rows were swapped. This changes the sign on the determinate of the matrix. det([17.]) $= -7$.

20. It looks like $R_1 + R_2 \to R_1$. This doesn't change anything about the determinate.

24. I used a computer to calculate the determinant of that matrix. $|24| = 11$. The matrix can be invertible because its determinant $\neq 0$. By the invertible matrix theorem the columns of this matrix must form a linearly independent set.

Section 4.1

1. a. If $\vec{u}$ and $\vec{v}$ are in V then $\vec{u} + \vec{v}$ must also be in V. If you use $\vec{e_1}$ and $\vec{e_2}$ those are two vectors that are just barely in V. Any positive combination of those vectors will also be in V.

   b. Let $\vec{u} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, c = -1$. The resulting vector $c\vec{u} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$ is not in V.

9. $H = \text{Span}\left\{ \begin{bmatrix} 30 \\ 90 \\ 60 \end{bmatrix} \right\}$ I don't understand theorem 1 well enough to not parrot it.

13. a. $\vec{w}$ is not in $\{\vec{v_1}, \vec{v_2}, \vec{v_3}\}$. There are 3 vectors in $\{\vec{v_1}, \vec{v_2}, \vec{v_3}\}$.

    b. There are infinitely many vectors in Span $\{\vec{v_1}, \vec{v_2}, \vec{v_3}\}$.

    c. Let $[A|\vec{w}] = \left[\begin{array}{ccc|c} 1 & 2 & 4 & 3 \\ 0 & 1 & 2 & 1 \\ -1 & 3 & 6 & 2 \end{array}\right] \xrightarrow{rref} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$ $\vec{w}$ is in the Span $\{\vec{v_1}, \vec{v_2}, \vec{v_3}\}$ because the system is consistent. $\vec{w}$ is in the subspace spanned by $\{\vec{v_1}, \vec{v_2}, \vec{v_3}\}$.

15. The zero vector is not in $\begin{bmatrix} 3a + b \\ 4 \\ a - 5b \end{bmatrix}$. Row 2 will always have the value 4 no matter the values of a and b.

17. I will cycle b through $-1, 0, 1$. $b = -1, a = 0, c = 0 \to \begin{bmatrix} 1 \\ -1 \\ 0 \\ -1 \end{bmatrix}$. $b = 0, a = 1, c = -1 \to \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$.

    $b = 1, a = 0, c = 1 \to \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$.

    Let $[A] = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \xrightarrow{rref} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$. These three vectors span a 3 dimensional subspace of $\mathbb{R}^4$.