Kyle Daling

MATH 204

Assignment 15

Section 1.9

35. If a transformation T maps $\mathbb{R}^n \to \mathbb{R}^m$ then the matrix of transformation must be $m \times n$. The matrix must also have a pivot in every row for the transformation to occur. Therefore, $m \leq n$. If T is one-to-one then the matrix of transformation must have a pivot in every column and the only way this is possible is if $m \geq n$.

Section 2.1

7. Matrix B must be 3×7 because A has 3 columns and the matrix AB has 7 columns.

9. If $k = 5$ then $AB = BA$. $AB = \begin{bmatrix} 8+15 & -10+5k \\ -12+3 & 15+k \end{bmatrix}$ and $BA = \begin{bmatrix} 8+15 & 20-5 \\ 6-3k & x+15 \end{bmatrix}$.

   $5k - 10 = 15 \Rightarrow 5k = 25 \Rightarrow k = 5$

   $6 - 3(5) = -9$ ✓and $15 + 5 = 5 + 15$ ✓

17. $\begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix} B = \begin{bmatrix} -1 & 2 & -1 \\ 6 & -9 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} A\vec{b_1} & A\vec{b_2} & A\vec{b_3} \end{bmatrix} = \begin{bmatrix} -1 & 2 & -1 \\ 6 & -9 & 3 \end{bmatrix}$

   So, $\begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 6 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -2 & -1 \\ -2 & 5 & 6 \end{bmatrix} \xrightarrow{rref} \begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & 4 \end{bmatrix} \therefore \vec{b_1} = \begin{bmatrix} 7 \\ 4 \end{bmatrix}$

   Similarly, $\begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2 \\ 5 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -2 & -2 \\ -2 & 5 & 5 \end{bmatrix} \xrightarrow{rref} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \therefore \vec{b_2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

20. If the second column of B is all zero, then the second column of AB must also be zero.

    Example: $A = \begin{bmatrix} 1 & 3 \\ 5 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \Rightarrow \text{AB} = \begin{bmatrix} 1+3 & 0 \\ 5+7 & 0 \end{bmatrix}$

21. If the last column of AB is zero and B has no zero columns, then A must have a zero column.

Section 2.2

1. $A_1^{-1} = \frac{1}{8(4)-6(5)} \begin{bmatrix} 4 & -6 \\ -5 & 8 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 4 & -6 \\ -5 & 8 \end{bmatrix}$

2. $A_2^{-1} = \frac{1}{3(4)-2(7)} \begin{bmatrix} 4 & -2 \\ -7 & 3 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -4 & 2 \\ 7 & -3 \end{bmatrix}$

5. $\vec{x} = \frac{1}{2} \begin{bmatrix} 4 & -6 \\ -5 & 8 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} \Rightarrow \vec{x} = \frac{1}{2} \begin{bmatrix} 8+6 \\ -10-8 \end{bmatrix} = \begin{bmatrix} 7 \\ -9 \end{bmatrix}$

27. a. We know $\text{row}_i(AB) = \text{row}_i(A)B$.

Let $B = I$

$\therefore \text{row}_i(AI) = \text{row}_i(A)I$

b. $A = \begin{bmatrix} \text{row}_1(A) \\ \text{row}_2(A) \\ \text{row}_3(A) \end{bmatrix}$ and $A_t = \begin{bmatrix} \text{row}_2(A) \\ \text{row}_1(A) \\ \text{row}_3(A) \end{bmatrix}$ and $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$EA = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \text{row}_1(A) \\ \text{row}_2(A) \\ \text{row}_3(A) \end{bmatrix} \Rightarrow \text{row}_1(A) \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + \text{row}_2(A) \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \text{row}_3(A) \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} \text{row}_2(A) \\ \text{row}_1(A) \\ \text{row}_3(A) \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} \text{row}_2(A) \\ \text{row}_1(A) \\ \text{row}_3(A) \end{bmatrix} = A_t$$

c. $A = \begin{bmatrix} \text{row}_1(A) \\ \text{row}_2(A) \\ \text{row}_3(A) \end{bmatrix}$ and $A_t = \begin{bmatrix} \text{row}_1(A) \\ \text{row}_2(A) \\ 5\text{row}_3(A) \end{bmatrix}$ and $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

$$EA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} \text{row}_1(A) \\ \text{row}_2(A) \\ \text{row}_3(A) \end{bmatrix} \Rightarrow \text{row}_1(A) \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \text{row}_2(A) \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + \text{row}_3(A) \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} \text{row}_1(A) \\ \text{row}_2(A) \\ 5\text{row}_3(A) \end{bmatrix} = A_t$$

28. $A = \begin{bmatrix} \text{row}_1(A) \\ \text{row}_2(A) \\ \text{row}_3(A) \end{bmatrix}$ and $A_t = \begin{bmatrix} \text{row}_1(A) \\ \text{row}_2(A) \\ \text{row}_3(A) - 4\text{row}_1(A) \end{bmatrix}$ and $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix}$

$$EA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix} \begin{bmatrix} \text{row}_1(A) \\ \text{row}_2(A) \\ \text{row}_3(A) \end{bmatrix} \Rightarrow \text{row}_1(A) \begin{bmatrix} 1 \\ 0 \\ -4 \end{bmatrix} + \text{row}_2(A) \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + \text{row}_3(A) \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} \text{row}_1(A) \\ \text{row}_2(A) \\ \text{row}_3(A) - 4\text{row}_1(A) \end{bmatrix} = A_t$$