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MATH 204

Assignment 14

Section 1.9

19. Let $A_{19} = \begin{bmatrix} 1 & -5 & 4 \\ 0 & 1 & -6 \end{bmatrix}$

27. (a) The mapping in 19 is not one-to-one because the columns of A_{19} cannot be linearly independent. The columns of A_{19} cannot be linearly independent because there are more column vectors than there are entries per column.

    (b) $A_{19} \xrightarrow{rref} = \begin{bmatrix} 1 & 0 & -26 \\ 0 & 1 & -6 \end{bmatrix}$ The mapping in 19 is onto because the columns of A_{19} span all of $\mathbb{R}^2$.

32. T maps $\mathbb{R}^n$ onto $\mathbb{R}^m$ if and only if A has $\underline{\ m\ }$ pivot columns. This statement is true because for a transformation to be onto, the matrix A must span all of $\mathbb{R}^m$. For a matrix to span all of $\mathbb{R}^m$ it must have a pivot in every row giving the matrix A, m total pivots. Therefor matrix A must have m pivot columns.

Supplementary Exercises

1. l. False, we only know existence because there is a pivot in every row. We don't know anything about pivots in the columns which would guarantee uniqueness.

   n. True, the matrices must be row equivalent which means they can be changed into one another by row operations.

   q. False, one vector could be the zero vector (which really is just any of the vectors multiplied by zero) which would make the vectors in the set linearly dependent.

   r. True, if 3 vectors are linearly independent, the vectors must have at least 3 entries per vector. This makes the 3 vectors elements of $\mathbb{R}^3$.

   s. False, it is impossible for 4 vectors to span $\mathbb{R}^5$. If the 4 vectors were put into a matrix with dimensions 5×4 it would be impossible for them to have a pivot in every row. Therefore it would be impossible for that matrix to span all of $\mathbb{R}^5$. Therefore it would be impossible for the column vectors of that matrix to span all of $\mathbb{R}^5$.

   u. False, we know nothing about their linear independence. $\vec{u}$ and $\vec{v}$ could be linearly dependent and $\vec{w}$ is a vector that is not in the span of $\vec{u}$ and $\vec{v}$ making $\vec{w}$ not a linear combination of $\vec{u}$ and $\vec{v}$.

   v. True, if $\vec{w}$ is a linear combination of vectors $\vec{u}$ and $\vec{v}$, then $\vec{u}$ and $\vec{v}$ must be linearly independent. Changing around the order will still have the vectors being linearly independent.

   w. True, $\vec{v_3}$ is not in the span of $\vec{v_1}$ and $\vec{v_2}$. With $\vec{v_1}$ and $\vec{v_2}$ not being multiples of one another, they too must be linearly independent. Adding an additional linearly independent vector to a linearly independent set of vectors will keep the set linearly independent.

x. False, a linear transformation could be a function, but it is also possible to have one input map to multiple outputs, which would make a linear transformation not a function.

y. True, the columns of a 6×5 matrix cannot span all of $\mathbb{R}^6$. Therefore a 6×5 matrix cannot map from $\mathbb{R}^5$ to $\mathbb{R}^6$.

z. False, the problem says nothing about the number of columns in the matrix. Column pivots are what determines whether or not a linear transformation will be one-to-one.

Section 2.1

1. c. $A \times C$ is undefined because A is a 2×3 matrix and C is a 2×2 matrix. The number of columns of A does not match the number of rows in C.

   d. $C \times D = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \times \begin{bmatrix} 3 & 5 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 3-2 & 1(5)+2(4) \\ -2(3)-1 & -2(5)+4 \end{bmatrix} = \begin{bmatrix} 1 & 13 \\ -7 & -6 \end{bmatrix}$

2. c. $C \times B = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \times \begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix} = \begin{bmatrix} 7+2 & -5-8 & 1-6 \\ -14+1 & 10-4 & -2-3 \end{bmatrix} = \begin{bmatrix} 9 & -13 & -5 \\ -13 & 6 & -5 \end{bmatrix}$

   d. $E \times B$ is undefined, the number of columns in E is not the same as the number of rows in B.

4. $A - (5I_3) = \begin{bmatrix} 9 & -1 & 3 \\ -8 & 7 & -6 \\ -4 & 1 & 8 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 4 & -1 & 3 \\ -8 & 2 & -6 \\ -4 & 1 & 3 \end{bmatrix}$

   $(5I_3)A = 5A = 5 \times \begin{bmatrix} 9 & -1 & 3 \\ -8 & 7 & -6 \\ -4 & 1 & 8 \end{bmatrix} = \begin{bmatrix} 45 & -5 & 15 \\ -40 & 35 & -30 \\ -20 & 5 & 40 \end{bmatrix}$

8. B must have 3 rows if BC is a 3×4 matrix.

10. $AB = \begin{bmatrix} 2 & -3 \\ -4 & 6 \end{bmatrix} \times \begin{bmatrix} 8 & 4 \\ 5 & 5 \end{bmatrix} = \begin{bmatrix} 16-15 & 8-15 \\ -32+30 & -16+30 \end{bmatrix} = \begin{bmatrix} 1 & -7 \\ -2 & 14 \end{bmatrix}$

    $AC = \begin{bmatrix} 2 & -3 \\ -4 & 6 \end{bmatrix} \times \begin{bmatrix} 5 & -2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 10-9 & -4-3 \\ -20+18 & 8+6 \end{bmatrix} = \begin{bmatrix} 1 & -7 \\ -2 & 14 \end{bmatrix} = AB$

12. Let $B = \begin{bmatrix} 6 & 2 \\ 3 & 1 \end{bmatrix} \Rightarrow AB = \begin{bmatrix} 3 & -6 \\ -1 & 2 \end{bmatrix} \times \begin{bmatrix} 6 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 18-18 & 6-6 \\ -6+6 & -2+2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

18. The first two columns of AB must also be equal because they will be multiplied by the same thing in A both times.