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MATH 204

Assignment 13

Section 1.8

17. $T(3\vec{u}) = 3T(\vec{u}) = 3\begin{bmatrix}2\\1\end{bmatrix} = \begin{bmatrix}6\\3\end{bmatrix}$, $T(2\vec{v}) = 2T(\vec{v}) = 2\begin{bmatrix}-1\\3\end{bmatrix} = \begin{bmatrix}-2\\6\end{bmatrix}$, and $T(3\vec{u}+2\vec{v}) = 3T(\vec{u}) + 2T(\vec{v}) = \begin{bmatrix}6\\3\end{bmatrix} + \begin{bmatrix}-2\\6\end{bmatrix} = \begin{bmatrix}4\\9\end{bmatrix}$

19. Let A_{19} be a 2×2 matrix.
$A_{19}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}2\\5\end{bmatrix}$ and $A_{19}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}-1\\6\end{bmatrix}$, $\therefore A_{19} = \begin{bmatrix}2 & -1\\5 & 6\end{bmatrix}$
$A_{19}\begin{bmatrix}5\\-3\end{bmatrix} = 5\begin{bmatrix}2\\5\end{bmatrix} - 3\begin{bmatrix}-1\\6\end{bmatrix} = \begin{bmatrix}13\\7\end{bmatrix}$
$A_{19}\begin{bmatrix}x_1\\x_2\end{bmatrix} = x_1\begin{bmatrix}2\\5\end{bmatrix} + x_2\begin{bmatrix}-1\\6\end{bmatrix} = \begin{bmatrix}2x_1 - x_2\\5x_1 + 6x_2\end{bmatrix}$

32. Let $A_{32} = \begin{bmatrix}4 & -2\\0 & |x_2|\end{bmatrix}$ The absolute value of x_2 is not a linear function. Therefore the transformation cannot be a linear transformation.

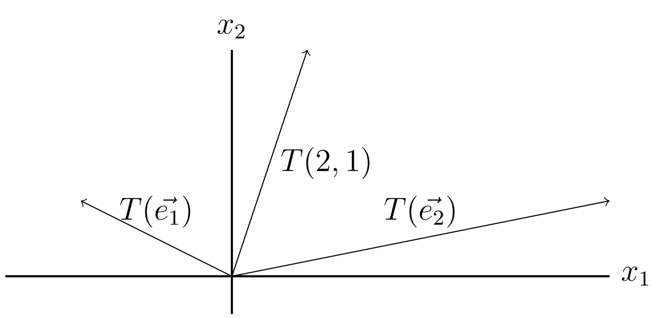
33. $T(x_1, x_2) = \begin{bmatrix}2x_1 - 3x_2\\x_1 + 4\\5x_2\end{bmatrix}$, $T(0,0) = \begin{bmatrix}0-0\\0+4\\0\end{bmatrix}$ Transforming the origin does not give the origin. Therefore this transformation cannot be a linear transformation.

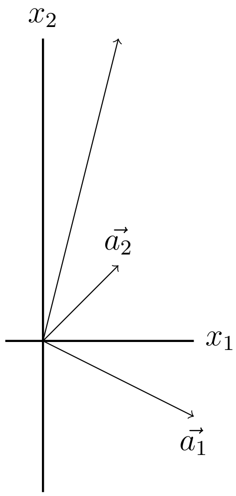
Section 1.9

3. Let $A_3 = \begin{bmatrix}0 & 1\\-1 & 0\end{bmatrix}$, $\begin{bmatrix}0 & 1\\-1 & 0\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}0\\-1\end{bmatrix}$, $\begin{bmatrix}0 & 1\\-1 & 0\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix}$

5. Let $A_5 = \begin{bmatrix}1 & 0\\-2 & 1\end{bmatrix}$, $\begin{bmatrix}1 & 0\\-2 & 1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1\\-2\end{bmatrix}$, $\begin{bmatrix}1 & 0\\-2 & 1\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}0\\1\end{bmatrix}$

8. Let $A_8 = \begin{bmatrix}0 & -1\\1 & 1\end{bmatrix}$, proof $\begin{bmatrix}0 & -1\\1 & 1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}0\\1\end{bmatrix}$ and $\begin{bmatrix}0 & 1\\1 & -1\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}-1\\1\end{bmatrix}$

13. 

14. 

20. $T(x_1, x_2, x_3, x_4) = \begin{bmatrix} 2 & 0 & 3 & -4 \end{bmatrix}$

22. $T(x_1, x_2) \Rightarrow \begin{bmatrix} 1 & -2 \\ -1 & 3 \\ 3 & -2 \end{bmatrix} \vec{x} = \begin{bmatrix} -1 \\ 4 \\ 9 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -2 & -1 \\ -1 & 3 & 4 \\ 3 & -2 & 9 \end{bmatrix} \xrightarrow{rref} \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow x_1 = 5$ and $x_2 = 3$

28. (a) The transformation in Exercise 14 is **one-to-one** because the two vectors are linearly independent.

    (b) The transformation in Exercise 14 is **onto** because the two vectors span all of $\mathbb{R}^2$ because they are linearly independent.

Section 1.10

9. Let $M = \begin{bmatrix} .95 & .04 \\ .05 & .96 \end{bmatrix}$ and $x_0 = \begin{bmatrix} 600000 \\ 400000 \end{bmatrix} \Rightarrow x_1 = \begin{bmatrix} 586000 \\ 414000 \end{bmatrix} \Rightarrow x_2 = \begin{bmatrix} 573260 \\ 426740 \end{bmatrix}$

12. $M = \begin{bmatrix} .97 & .05 & .10 \\ 0 & .9 & .05 \\ .03 & .05 & .85 \end{bmatrix}$ and $x_{\text{Monday}} = \begin{bmatrix} 304 \\ 48 \\ 98 \end{bmatrix} \Rightarrow x_{\text{Tuesday}} = \begin{bmatrix} .97 & .05 & .10 \\ 0 & .9 & .05 \\ .03 & .05 & .85 \end{bmatrix} \times \begin{bmatrix} 304 \\ 48 \\ 98 \end{bmatrix} = \begin{bmatrix} 307.08 \\ 48.1 \\ 94.82 \end{bmatrix} \Rightarrow x_{\text{Wednesday}} = M \times x_{\text{Tuesday}} = \begin{bmatrix} 309.555 \\ 47.931 \\ 90.5144 \end{bmatrix}$

Supplementary Exercises

1. e. False, the solution sets can row equivalent, but not necessarily exactly the same.

   f. True, if a non-homogeneous system is consistent and has infinite solution it must have a pivotless column. If put into a homogeneous system, the homogeneous system must also have a pivotless column forcing the system have infinite solutions.

   g. False, the matrix A must be consistent for **all** $\vec{b}$ in $\mathbb{R}^m$.

h. False, what are the pivots. There could be a pivot in the augmented column which would make the system inconsistent.

j. True, there must be a pivot in every column for a homogeneous system to have only the trivial solution.

k. True, this is equivalent to saying that A spans all of $\mathbb{R}^m$ which requires a pivot in every row.

Handout

3. (a) False, if the vectors given by the columns of A are linearly independent, they can span all of $\mathbb{R}^2$. It is possible for A to have a pivot in every column.

   (b) False, homogeneous systems are guaranteed to always be consistent because it is impossible for them to have a pivot in the augmented column. It is possible for a pivot to be in the augmented column in the system $A\vec{x} = \vec{b}$ which would make the system inconsistent.