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MATH 204

Assignment 11

Section 1.7

27. A $7x5$ matrix must have 5 pivot columns to make it linearly independent. This is the only way the system can have only the trivial solution and no free variables.

36. False, by counterexample $\vec{v_1} = \vec{v_2} = \vec{v_4} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ and $\vec{v_3} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$. $\vec{v_3}$ is not in the span of $\{\vec{v_1}, \vec{v_2}, \vec{v_4}\}$ and $\{\vec{v_1}, \vec{v_2}, \vec{v_3}, \vec{v_4}\}$ is not linearly independent.

37. True, two of the vectors in $\{\vec{v_1}, \vec{v_2}, \vec{v_3}, \vec{v_4}\}$ are already linearly dependent by the definition in the problem. Adding $\vec{v_4}$ to the set can't make two linearly dependent vectors linearly independent.

40. If a $m \times n$ matrix A has n pivots it must have a pivot in every column. The equation $A\vec{x} = \vec{0} \in \mathbb{R}^m$ must have only the trivial solution because the system has no free variables.

Section 1.8

1. $A\vec{u} = \begin{bmatrix} 2 \\ -6 \end{bmatrix}$, $A\vec{v} = \begin{bmatrix} 2a \\ 2b \end{bmatrix}$

4. $A\vec{x} = \vec{b} \Rightarrow \begin{bmatrix} 1 & -3 & 2 \\ 0 & 1 & -4 \\ 3 & -5 & -9 \end{bmatrix} \vec{x} = \begin{bmatrix} 6 \\ -7 \\ -9 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -3 & 2 & 6 \\ 0 & 1 & -4 & -7 \\ 3 & -5 & -9 & -9 \end{bmatrix} \xrightarrow{rref} \begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 \end{bmatrix} \Rightarrow x_1 = -5$ and $x_2 = -3$ and $x_3 = 1$. This solution is unique because there is a pivot in every column.

5. $A\vec{x} = \vec{b} \Rightarrow \begin{bmatrix} 1 & -5 & -7 & -2 \\ -3 & 7 & 5 & -2 \end{bmatrix} \xrightarrow{rref} \begin{bmatrix} 1 & 0 & 3 & 3 \\ 0 & 1 & 2 & 1 \end{bmatrix} \Rightarrow x_1 = 3 - 3x_3$ and $x_2 = 1 - 2x_3$ and $x_3 = x_3$. This solution is not unique because there is not a pivot in every column of A making x_3 free.

7. $a = 5$ and $b = 6$ because $\vec{x}$ must be in $\mathbb{R}^5$ so it can map A into $\mathbb{R}^6$.

8. Matrix A must be 5×4 because $\vec{x}$ must be in $\mathbb{R}^4$ so it can map A into $\mathbb{R}^5$.

9. $A\vec{x} = \vec{0} \Rightarrow \begin{bmatrix} 1 & -4 & 7 & -5 & 0 \\ 0 & 1 & -4 & 3 & 0 \\ 2 & -6 & 6 & -4 & 0 \end{bmatrix} \xrightarrow{rref} \begin{bmatrix} 1 & 0 & -9 & 7 & 0 \\ 0 & 1 & -4 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow x_1 = 9x_3 - 7x_4$ and $x_2 = 4x_3 - 3x_4$ and $x_3 = x_3$ and $x_4 = x_4 \Rightarrow \vec{x} = x_3 \begin{bmatrix} 9 \\ 4 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -7 \\ -3 \\ 0 \\ 1 \end{bmatrix}$

10. $A\vec{x} = \vec{0} \Rightarrow \begin{bmatrix} 1 & 3 & 9 & 2 & 0 \\ 1 & 0 & 3 & -4 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ -2 & 3 & 0 & 5 & 0 \end{bmatrix} \xrightarrow{rref} \begin{bmatrix} 1 & 0 & 3 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow x_1 = -3x_3$ and $x_2 = -2x_3$ and $x_3 = x_3$ and $x_4 = 0 \Rightarrow \vec{x} = x_3 \begin{bmatrix} -3 \\ -2 \\ 1 \\ 0 \end{bmatrix}$.

11. $\begin{bmatrix} 1 & -4 & 7 & -5 & -1 \\ 0 & 1 & -4 & 3 & 1 \\ 2 & -6 & 6 & -4 & 0 \end{bmatrix} \xrightarrow{rref} \begin{bmatrix} 1 & 0 & -9 & 7 & 3 \\ 0 & 1 & -4 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Yes, $\vec{b}$ is in the linear transformation because the system is consistent.

12. $\begin{bmatrix} 1 & 3 & 9 & 2 & -1 \\ 1 & 0 & 3 & -4 & 3 \\ 0 & 1 & 2 & 3 & -1 \\ -2 & 3 & 0 & 5 & 4 \end{bmatrix} \xrightarrow{rref} \begin{bmatrix} 1 & 0 & 3 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

No, $\vec{b}$ is not in the range of the linear transformation because there is a pivot in the augmented column making the system inconsistent.