Kyle Daling

MATH 204

Assignment 10

5. $\begin{bmatrix} 0 & -8 & 5 & 0 \\ 3 & -7 & 4 & 0 \\ -1 & 5 & -4 & 0 \\ 1 & -3 & 2 & 0 \end{bmatrix} \xrightarrow{rref} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

   The columns of the matrix form a linearly independent set because only the trivial solution exists. There are no free variables in the system.

6. $\begin{bmatrix} -4 & 3 & 0 & 0 \\ 0 & -1 & 4 & 0 \\ 1 & 0 & 3 & 0 \\ 5 & 4 & 6 & 0 \end{bmatrix} \xrightarrow{rref} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

   The columns of the matrix form a linearly independent set because only the trivial solution exists. There are no free variables in the system.

14. $\begin{bmatrix} 1 & -5 & 1 & 0 \\ -1 & 7 & 1 & 0 \\ -3 & 8 & h & 0 \end{bmatrix} \xrightarrow{R_1+R_2 \to R_2} \begin{bmatrix} 1 & -5 & 1 & 0 \\ 0 & 2 & 2 & 0 \\ -3 & 8 & h & 0 \end{bmatrix} \xrightarrow{3R_1+R_3 \to R_3} \begin{bmatrix} 1 & -5 & 1 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & -7 & h+3 & 0 \end{bmatrix} \xrightarrow{\frac{7}{2}R_2+R_3 \to R_3}$

    $\begin{bmatrix} 1 & -5 & 1 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & 0 & h+10 & 0 \end{bmatrix}$

    $h = 10$ makes the vectors linearly dependent because it prevents a pivot from being in column 3 and allows x_3 to be a free variable.

17. The vectors are linearly dependent. The zero vector makes all other vectors dependent.

18. The vectors must be linearly dependent. There are only 2 rows, which means there can be at most 2 pivots. This leaves two variables free.

29. $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

30. a. If A is an $m \times n$ matrix, then the columns of A are linearly independent if and only if A has $\underline{n}$ pivot columns.

    b. If A didn't have n pivot columns, at least one variable in A would be free. This would make the set of columns of A linearly dependent as non-trivial solutions would exist.

31. $x_1 = 1$ and $x_2 = 1$ and $x_3 = -1 \Rightarrow 1 \begin{bmatrix} 2 \\ -5 \\ -3 \\ 1 \end{bmatrix} + 1 \begin{bmatrix} 3 \\ 1 \\ -1 \\ 0 \end{bmatrix} - 1 \begin{bmatrix} 5 \\ -4 \\ -4 \\ 1 \end{bmatrix} = \vec{0} \in \mathbb{R}^4$

33. True, if the vectors were linearly independent then they would all appear in the equation for $\vec{v_3}$. $\vec{v_4}$ doesn't affect the equation for $\vec{v_3}$ so it must be free.

34. True, the $\vec{0}$ vector is linearly dependent of all other vectors.

35. True, the vectors can't be collinear and neither can be the zero vector (because zero is a scalar) so they must be independent.

38. True, there must be 4 pivots to make the 4 vectors linearly independent. If one vector was removed, the resulting set of vectors would only span a 3 dimensional subspace of $\mathbb{R}^4$ but because they would only have 3 pivots. Those 3 vectors must be linearly independent so they can span all of that 3 dimensional subspace.

39. The columns of A must be linearly independent because if there was a free variable there would be infinite solutions. Since there is only one solution, the vectors must be linearly independent.