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MATH 204

Assignment 5

Section 1.3

6.

$$\begin{aligned} -2x_1 + 8x_2 + x_3 &= 0 \\ 3x_1 + 5x_2 - 6x_3 &= 0 \end{aligned}$$

8. All vectors in $\mathbb{R}^2$ can be expressed as a linear combination of these particular $\vec{u}$ and $\vec{v}$.

$$\begin{aligned} \vec{w} &= -\vec{u} + 2\vec{v} \\ \vec{x} &= -2\vec{u} + 2\vec{v} \\ \vec{y} &= -2\vec{u} + 3.5\vec{v} \\ \vec{z} &= 4\vec{u} + 3\vec{v} \end{aligned}$$

11. $\begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} x_2 + \begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix} x_3 = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 & 5 & 2 \\ -2 & 1 & -6 & -1 \\ 0 & 2 & 8 & 6 \end{bmatrix} \xrightarrow{rref} \begin{bmatrix} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The system of equations is consistent so $\vec{b}$ must be a linear combination of $\vec{a_1}$ and $\vec{a_2}$ and $\vec{a_3}$.

12. $\begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix} x_2 + \begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix} x_3 = \begin{bmatrix} -5 \\ 11 \\ -7 \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 & 2 & -5 \\ -2 & 5 & 0 & 11 \\ 2 & 5 & 8 & -7 \end{bmatrix} \xrightarrow{rref} \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & \frac{4}{5} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The system is inconsistent, $\vec{b}$ cannot be a linear combination of $\vec{a_1}$ and $\vec{a_2}$ and $\vec{a_3}$.

13. $\begin{bmatrix} 1 & -4 & 2 & 3 \\ 0 & 3 & 5 & -7 \\ -2 & 8 & -4 & -3 \end{bmatrix} \xrightarrow{rref} \begin{bmatrix} 1 & 0 & \frac{26}{3} & 0 \\ 0 & 1 & \frac{5}{3} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

The system is inconsistent $\vec{b}$ cannot be a linear combination of $\vec{a_1}$ and $\vec{a_2}$ and $\vec{a_3}$.

14. $\begin{bmatrix} 1 & -2 & -6 & 11 \\ 0 & 3 & 7 & -5 \\ 1 & -2 & 5 & 9 \end{bmatrix} \xrightarrow{rref} \begin{bmatrix} 1 & 0 & 0 & \frac{245}{33} \\ 0 & 1 & 0 & \frac{-41}{33} \\ 0 & 0 & 1 & \frac{-2}{11} \end{bmatrix}$

The system of equations is consistent so $\vec{b}$ must be a linear combination of $\vec{a_1}$ and $\vec{a_2}$ and $\vec{a_3}$.

15.

$$0\vec{v_1} + 0\vec{v_2} = \vec{0}$$

$$-1\vec{v_1} + 0\vec{v_2} = \begin{bmatrix} -7 \\ -1 \\ 6 \end{bmatrix}$$

$$0\vec{v_1} - 1\vec{v_2} = \begin{bmatrix} 5 \\ -3 \\ 0 \end{bmatrix}$$

17. $\begin{bmatrix} 1 & -2 & 4 \\ 4 & -3 & 1 \\ -2 & 7 & h \end{bmatrix} \xrightarrow{-4R_1+R_2 \to R_2} \begin{bmatrix} 1 & -2 & 4 \\ 0 & 5 & -15 \\ -2 & 7 & h \end{bmatrix} \xrightarrow{2R_1+R_3 \to R_3} \begin{bmatrix} 1 & -2 & 4 \\ 0 & 5 & -15 \\ 0 & 3 & h+8 \end{bmatrix} \xrightarrow{\frac{1}{5}R_2 \to R_2}$

$\begin{bmatrix} 1 & -2 & 4 \\ 0 & 1 & -3 \\ 0 & 3 & h+8 \end{bmatrix} \xrightarrow{-3R_2+R_3 \to R_3} \begin{bmatrix} 1 & -2 & 4 \\ 0 & 1 & -3 \\ 0 & 0 & h+17 \end{bmatrix}$

To make the system consistent, $h \neq -17$. This would allow $\vec{b}$ to be spanned by $\vec{a_1}$ and $\vec{a_2}$.

19. Span$\{\vec{v_1}, \vec{v_2}\}$ a line made of multiples of $\vec{v_1}$.

26. a. $\begin{bmatrix} 2 & 0 & 6 & 10 \\ -1 & 8 & 5 & 3 \\ 1 & -2 & 1 & 3 \end{bmatrix} \xrightarrow{rref} \begin{bmatrix} 1 & 0 & 3 & 5 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Since the system is consistent, $\vec{b}$ is in W.

b. $\begin{bmatrix} 2 & 0 & 6 \\ -1 & 8 & 5 \\ 1 & -2 & 1 \end{bmatrix} \xrightarrow{rref} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

Since the system is consistent, the third column of A is in W.

Section 1.4

1. Undefined, the dimensions of the 3×2 matrix are incorrect for multiplying by a 3×1 matrix.

2. Undefined, the dimensions of the 2×1 matrix are incorrect for multiplying by a 2×1 matrix.

3. $2 \begin{bmatrix} 6 \\ -4 \\ 7 \end{bmatrix} - 3 \begin{bmatrix} 5 \\ -3 \\ 6 \end{bmatrix} = \begin{bmatrix} 12 \\ -8 \\ 14 \end{bmatrix} + \begin{bmatrix} -15 \\ 9 \\ -18 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ -4 \end{bmatrix}$

4. $1\begin{bmatrix}8\\5\end{bmatrix}+1\begin{bmatrix}3\\1\end{bmatrix}+1\begin{bmatrix}-4\\2\end{bmatrix}=\begin{bmatrix}7\\8\end{bmatrix}$

Supplementary Exercises

5. a. $\begin{bmatrix}1 & 3 & k\\4 & h & 8\end{bmatrix}\xrightarrow{-4R_1+R_2\to R_2}\begin{bmatrix}1 & 3 & k\\0 & h-12 & 8-4k\end{bmatrix}$

Infinite Solutions : $h-12=0$ and $8-4k=0 \Rightarrow h=12$ and $k=2$
No solution : $h=12$ and $k\neq 2$
One Solution : $h\neq 12$ and $k\in\mathbb{R}$

b. $\begin{bmatrix}-2 & h & 1\\6 & k & -2\end{bmatrix}\xrightarrow{3R_1+R_2\to R_2}\begin{bmatrix}-2 & h & 1\\0 & 3h+k & 1\end{bmatrix}$

Infinite Solutions : Not possible
No solution : $3h+k=0 \Rightarrow k=-3h$
One Solution : $k\neq -3h$