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MATH 204

Assignment #3

Section 1.2

4. $$\begin{bmatrix} \textcircled{1} & 3 & 5 & 7 \\ 3 & \textcircled{5} & 7 & 9 \\ 5 & 7 & 9 & \textcircled{1} \end{bmatrix} \xrightarrow{-3R_1+R_2\to R_2} \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & -4 & -8 & -12 \\ 5 & 7 & 9 & 1 \end{bmatrix} \xrightarrow{-5R_1+R_3\to R_3} \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & -4 & -8 & -12 \\ 0 & -8 & -16 & -34 \end{bmatrix}$$

$$\xrightarrow{-2R_2+R_3\to R_3} \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & -4 & -8 & -12 \\ 0 & 0 & 0 & -10 \end{bmatrix} \xrightarrow{-\frac{1}{4}R_2\to R_2} \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & -10 \end{bmatrix} \xrightarrow{-\frac{1}{10}R_3\to R_3} \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{-3R_3+R_2\to R_2} \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{-7R_3+R_1\to R_1} \begin{bmatrix} 1 & 3 & 5 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{-3R_2+R_1\to R_1} \begin{bmatrix} \textcircled{1} & 0 & -1 & 0 \\ 0 & \textcircled{1} & 2 & 0 \\ 0 & 0 & 0 & \textcircled{1} \end{bmatrix}$$

Pivot columns are columns 1, 2, and 4.

6. $$\begin{bmatrix} \blacksquare & * \\ 0 & \blacksquare \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & \blacksquare \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} \blacksquare & * \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

10. $$\begin{bmatrix} 1 & -2 & -1 & 3 \\ 3 & -6 & -2 & 2 \end{bmatrix} \xrightarrow{-3R_1+R_2\to R_2} \begin{bmatrix} 1 & -2 & -1 & 3 \\ 0 & 0 & 1 & -7 \end{bmatrix} \xrightarrow{R_2+R_1\to R_1} \begin{bmatrix} 1 & -2 & 0 & -4 \\ 0 & 0 & 1 & -7 \end{bmatrix}$$

$x_1 = 2x_2 - 4$, $x_2 = x_2$, $x_3 = -7$

12. $$\begin{bmatrix} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ -1 & 7 & -4 & 2 & 7 \end{bmatrix} \xrightarrow{R_1+R_3\to R_3} \begin{bmatrix} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ 0 & 0 & -4 & 8 & 12 \end{bmatrix} \xrightarrow{4R_2+R_3\to R_3}$$

$$\begin{bmatrix} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_1 = 5 - 6x_4 + 7x_2$, $x_2 = x_2$, $x_3 = 2x_4 - 3$, $x_4 = x_4$

15. a. System is consistent with a unique solution. There are no pivots in the augmented column.

    b. The system is not consistent, there is a pivot in the augmented column.

17. h must be equal to 3.5 to make the system consistent. The two rows in the matrix are parallel to one another, meaning the only way they can have a consistent solution is if they are the same line.

18. $\begin{bmatrix} 1 & -3 & -2 \\ 5 & h & -7 \end{bmatrix} \xrightarrow{-5R_1+R_2 \to R_2} \begin{bmatrix} 1 & -3 & -2 \\ 0 & (15+h) & 3 \end{bmatrix}$ Any value of h makes the system consistent.
$(15+h)x_2 = 3$

20. (a) No Solution: $h = 9$, $k \neq 6$. Lines with the same slope but different intercepts.
    (b) Unique Solution: $h \neq 9$, $k \in \mathbb{R}$
    (c) Infinite Solutions: $h = 9$, $k = 6$. These values make the lines have the same slope and intercept.

29. Having more variables than equations forces at least one of the variables to be free. A free variable creates an infinite amount of solutions.

31. Yes, a overdetermined system can be consistent. When fully reduced, the augmented matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 11 & 13 & 17 \end{bmatrix}$ gives the matrix $\begin{bmatrix} 1 & 0 & \frac{-5}{9} \\ 0 & 1 & \frac{16}{9} \\ 0 & 0 & 0 \end{bmatrix}$ which has a unique solution.

Supplementary Excercises

1. a. False, matrices can be reduced into any number of other matrices in echelon form.
   b. False, systems of any number of linear equations can have infinite or zero solutions.
   c. True, if the system has a solution it must be consistent. If the system is consistent and has two solutions, then the system must have infinitely many solutions.
   d. False, if there is a pivot in the augmented column, the system is inconsistent and has no solution despite having no free variables.

Handout Exercises

1. (a) True, row operations don't change the information in the matrix, they just change how it looks.
   (b) False, rows can only be multiplied by a non-zero constant.
   (c) True, if the matrices are row equivalent that means they can be transformed into one another by row scaling operations.