CSCI 301 Formal languages and Functional Programming
Assignment 7
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Section 11.1

4. The relation is **reflexive**, **symmetric**, and **transitive**.

Section 11.2

8. To prove reflexive, assume $x\text{R}x$
   Therefore, $x^2 + y^2 = x^2 + x^2 = 2x^2$
   $= 2(z)$ for $z \in \mathbb{Z}, z = x^2$

   To prove symmetry, assume $x\text{R}y$ and $x^2 + y^2$ is even
   $x^2 + y^2 = 2a$ for $a \in \mathbb{Z}$
   $= y^2 + x^2$
   $y^2 + x^2$ will be even as well as it is just a rearrangement of the terms
   $\therefore x\text{R}y$ and $y\text{R}x$

   To prove transitivity assume $x\text{R}y$ and $y\text{R}z$
   $x^2 + y^2 = 2b$ for $b \in \mathbb{Z}$
   $y^2 + z^2 = 2c$ for $c \in \mathbb{Z}$
   $x^2 + 2y^2 + z^2 = 2b + 2c$
   $x^2 + z^2 = 2(b + c - y)$
   $\therefore x^2 + z^2$ is even so $x\text{R}z$
   We have thus proved R is an equivalence relation because it is reflexive, symmetric, and transitive.
   Equivalence classes:
   $[0] = \{x \in \mathbb{Z} : x\text{R}0\} = \{x \in \mathbb{Z} : x^2 + 0^2 \text{ is even}\} = \{x \in \mathbb{Z} : x^2 \text{ is even}\} = \{x \in \mathbb{Z} : x \text{ is even}\}$
   $[1] = \{x \in \mathbb{Z} : x\text{R}1\} = \{x \in \mathbb{Z} : x^2 + 1^2 \text{ is even}\} = \{x \in \mathbb{Z} : x^2 \text{ is odd}\} = \{x \in \mathbb{Z} : x \text{ is odd}\}$

   $\therefore$ There are two equivalence classes for x, one containing all even integers, the other containing all odd integers.

Section 12.1

6. **Domain:** All integers

Codomain: All integers

Range: All integers

$f(10) = (10, 4 \times 10 + 5) = (10, 45)$

Section 12.2

6. **Injective**:
Suppose via contrapositive $x, y \in$ A and $f(x, y) = f(y, x)$
$f(x, y) = 3x - 4y$
$f(y, x) = 3y - 4x$
$3x - 4y = 3y - 4x$
$3x + 4x = 3y + 4y$
$7x = 7y$
$\therefore x = y$
Therefore $f(m, n)$ is injective

Surjective:
Suppose $b \in$ B
$f(c, d) = b$
$3c - 4d = b$

As $3c$ can be odd or even and $4d$ is even, any integer b can be created by changing the integers c and d.

Section 12.4

4. $g \circ f = \{(a, a), (b, a), (c, a)\}$
$f \circ g = \{(a, c), (b, c), (c, c)\}$