CSCI 301 Formal languages and Functional Programming
Assignment 6
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2. Proof by induction

   Base Case: for $n = 1$
   $\frac{n(n+1)(2n+1)}{6} = \frac{1(2)(3)}{6} = \frac{6}{6} = 1 = 1^2 = n^2$
   $\therefore$ true for $n = 1$
   Induction Step: Assume true for $n = k$
   $1^2 + 2^2 + \ldots + (k)^2 = \sum_{i=1}^{k} i^2 = \frac{k(k+1)(2k+1)}{6}$
   Prove true for $n = k + 1$
   $$\sum_{i=1}^{k+1} i^2 = (k+1)^2 + \sum_{i=1}^{k} i^2$$
   $$= (k+1)^2 + \frac{k(k+1)(2k+1)}{6}$$
   $$= \frac{6(k+1)^2}{6} + \frac{k(k+1)(2k+1)}{6}$$
   $$= \frac{6(k+1)^2+k(k+1)(2k+1)}{6}$$
   $$= \frac{(k+1)(6(k+1)+k(2k+1)}{6}$$
   $$= \frac{(k+1)(6k+6+2k^2+k)}{6}$$
   $$= \frac{(k+1)(2k^2+7k+6)}{6}$$
   $$= \frac{(k+1)(2k+3)(2k+2)}{6}$$
   $$= \frac{(k+1)(2k+3)(2k+2)}{6}$$
   $$= \frac{(k+1)(2k+2)(2k+3)}{6}$$
   $$= \frac{(k+1)(k+2)(2k+2+1)}{6}$$
   $$= \frac{(k+1)((k+1)+1)(2(k+1)+1)}{6}$$
   $$\therefore \sum_{i=1}^{k+1} i^2 = \frac{(k+1)((k+1)+1)(2(k+1)+1)}{6}$$
   Therefore this relationship is true for all natural numbers $n > 0$

4. Proof by induction of $1 \times 2 + 2 \times 3 + \ldots + n(n+1) = \frac{n(n+1)(n+2)}{3}$
   Base Case: $n = 1$
   $1(1+1) = \frac{1(1+1)(1+2)}{3} = 2$
   $\therefore$ This is true for $n = 1$
   Induction Step: Assume true for $n = k$
   $1 \times 2 + 2 \times 3 + \ldots + k(k+1) = \sum_{i=1}^{k} i(i+1) = \frac{k(k+1)(k+2)}{3}$
   $$\sum_{i=1}^{k+1} i(i+1) = (k+1)(k+1+1) + \frac{k(k+1)(k+2)}{3}$$
   $$= \frac{3(k+1)(k+2)}{3} + \frac{k(k+1)(k+2)}{3} = \frac{3(k+1)(k+2)+k(k+1)(k+2)}{3}$$

$= \frac{(k+1)(k+2)(3+k)}{3}$
$= \frac{(k+1)(k+2)(k+3)}{3}$
$= \frac{(k+1)((k+1)+1)((k+1)+2)}{3}$
$\therefore \sum_{i=1}^{k+1} i(i+1) = \frac{(k+1)((k+1)+1)((k+1)+2)}{3}$
Therefore this is true for all natural numbers $n > 0$

6. Proof by induction
Base Case: $n = 1$
$\sum_{i=1}^{1}(8i-5) = 4(1)^2 - 1 = 3$
$\therefore$ This relationship is true for the base case $n = 1$.
Induction Step: Assume true for $n = k$
$\sum_{i=1}^{k}(8i-5) = 4(k)^2 - k$
Prove true for $n = k + 1$
$\sum_{i=1}^{k+1}(8i-5) = (8(k+1)-5) + \sum_{i=1}^{k}(8i-5)$
$= (8k+8)-5) + 4k^2 - k$
$= (8k+3) + 4k^2 - k$
$= 4k^2 + 7k + 3$
$= 4k^2 + 7k + k + 3 + 1 - k - 1$
$= 4(k^2 + 2k + 1) - (k+1)$
$= 4(k+1)^2 - (k+1)$
$\therefore$ This relationship is true for $n = k + 1$.
Therefore this relationship is true for all natural numbers $n > 0$.

8. Proof by induction
Base Case: $n = 1$
$\frac{1}{(1+1)!} = 1 - \frac{1}{(1+1)!}$
$\frac{1}{(2)!} = 1 - \frac{1}{(2)!} = \frac{1}{2}$
$\therefore$ This relationship is true for $n = 1$.
Induction Step: Assume true for $n = k$.
$\frac{1}{(1+1)!} + \frac{2}{(2+1)!} + \ldots + \frac{k}{(k+1)!} = \sum_{i=1}^{k} \frac{i}{(i+1)!} = 1 - \frac{1}{(k+1)!}$
Prove true for $n = (k + 1)$.
$\sum_{i=1}^{k+1} \frac{i}{(i+1)!} = \frac{(k+1)}{((k+1)+1)!} + \sum_{i=1}^{k} \frac{i}{(i+1)!}$
$= \frac{(k+1)}{(k+2)!} + (1 - \frac{1}{(k+1)!})$
$= 1 + \frac{(k+1)}{(k+2)!} - \frac{1}{(k+1)!}$
$= 1 + \frac{(k+1)(k+1)!}{(k+2)!(k+1)!} - \frac{1(k+2)!}{(k+1)!(k+2)!}$
$= 1 + \frac{(k+1)(k+1)! - (k+2)!}{(k+1)!(k+2)!}$
$= 1 + \frac{(k+1)!(k+1-k-2)}{(k+1)!(k+2)!}$

$= 1 + \frac{(k+1)!(1-2)}{(k+1)!(k+2)!}$
$= 1 + \frac{-1}{(k+2)!}$
$= 1 - \frac{1}{(k+2)!}$
$= 1 - \frac{1}{((k+1)+1)!}$
$\therefore$ This relationship is true for $n = (k+1)$.
Therefore this relationship is true for all natural numbers $n > 0$.

10. Proof by induction
Base Case: Assume $n = 0$
$(5^{2\times 0} - 1) = 0$
$0 = 3a$ for $a \in \mathbb{Z} \wedge a = 0$
$\therefore$ This relationship is true for $n = 0$.
Induction Step: Assume true for $n = k$.
$(5^{2\times k} - 1) = 3b$ for $b \in \mathbb{Z}$
$5^{2k} = 3b + 1$
Prove true for $n = (k+1)$
$5^{2(k+1)} - 1$
$= 5^{2k+2} - 1$
$= 5^{2k} \times 5^2 - 1$
$= 5^2 \times (3b+1) - 1$
$= 25 \times 3b + 25 - 1$
$= 25 \times 3b + 24$
$= 3(25b + 8)$
$5^{2(k+1)} - 1 = 3c$ for $c \in \mathbb{Z} \wedge c = (25b + 8)$
$\therefore$ This relationship is true for $n = (k+1)$.
Therefore this relationship is true for all integers $n \geq 0$