CSCI 301 Formal languages and Functional Programming
Assignment 5
Winter, 2016

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Section 3.1

8. (a) Lists of length 5 with at least one letter repeated
   All possible lists of length $5 = 5^5$
   Lists of length 5 with no letters repeated $= 5! = 5 \times 4 \times 3 \times 2 \times 1$
   Lists of length 5 with at least one letter repeated $= 5^5 - 5!$

   (b) Lists of length 6 with at least one letter repeated
   All possible lists of length $6 = 5^6$
   But because there are only 5 elements to choose from, one letter will always be repeated.
   So, lists of length 6 with at least one letter repeated $= 5^6$

Section 3.3

12. Repeats: **no**, Ordered: **no** $\Rightarrow$ subsets
    Number of ways to choose Blue Team $= \binom{21}{11}$
    Number of ways to choose Red Team $= \binom{21}{10}$
    $\binom{21}{10} + \binom{21}{11}$

Section 3.5

4. Lists $\Rightarrow$ Ordered: **yes**, Repetition: **yes**
   (a) $5 \times 6 \times 6 \times 5 = 5^2 * 6^2$
   (b) $2 * 6$
   (c) $3 * 6^2$

Chapter 6

8. Assume by contradiction that $(a^2 + b^2 = c^2) \wedge (a \text{ and } b \text{ are odd})$

$a = 2x + 1$ for $x \in \mathbb{Z}$
$b = 2y + 1$ for $y \in \mathbb{Z}$

$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= (2x+1)^2 + (2y+1)^2 \\ &= 4x^2 + 4x + 1 + 4y^2 + 4y + 1 \\ &= 4x^2 + 4y^2 + 4x + 4y + 2 \\ &= 2(2x^2 + 2y^2 + 2x + 2y + 1) \\ &= 2z \text{ for } z \in \mathbb{Z} \wedge z = 2x^2 + 2y^2 + 2x + 2y + 1 \end{aligned}$$

However, it is possible for c^2 to be odd, therefore c^2 is both odd and even.
Therefore we have proved the original statement via contradiction.

10. Assume by contradiction that integers a, b exist for which $21a + 30b = 1$
Divide both sides of the equation by 3
$7a + 10b = \frac{1}{3}$
a and b are integers, so their sum will also be an integer
$\therefore \frac{1}{3}$ is an integer
Since $\frac{1}{3}$ is not an integer, we have proved the original statement via contradiction.